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Time-Delay Effects on Linear/Nonlinear Feedback Control of Simple Aeroelastic Systems

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A study of the effects of time delay on the linear/nonlinear feedback control of two-dimensional lifting surfaces in an incompressible flowfield is presented. Specifically, the case of a one-degree-of-freedom system is considered in detail, and in that context, both the structural and the unsteady aerodynamics models are assumed to be linear. The study of the stability/instability behavior of nonlinear feedback time-delay closed-loop aeroelastic systems is carried out via Pontryagin's approach in conjunction with Stépán's theorems and the associated aeroelastic Volterra kernels. Results are presented, and conclusions on the implication of the time delay on feedback control are highlighted.

Nomenclature

a	=	dimensionless elastic axis position measured from the midchord, positive aft
b	=	semichord
$C(s)$	=	Theodorsen's function
$C_{L\alpha}$	=	lift–curve slope, 2π
c, k	=	damping and stiffness parameters of one-degree-of-freedom plunging airfoil respectively
c_h, c_α, c_β	=	damping parameters in plunging, pitching, and flapping, respectively
e	=	dimensionless leading edge flap position measured from the midchord, positive aft
$\boldsymbol{F}_a, \boldsymbol{F}_b$	=	aerodynamic and time-dependent load vectors
$\boldsymbol{F}_c$	=	control force
$\boldsymbol{G}$	=	control input matrix
$\boldsymbol{g}_p, \boldsymbol{g}_v, \boldsymbol{g}_{\mathrm{nc}}$	=	proportional, velocity, and nonlinear feedback control gain matrices
$g_p, g_v, g_{\mathrm{nc}}$; $\mathsf{g}_p, \mathsf{g}_v, \mathsf{g}_{\mathrm{nc}}$	=	proportional, velocity, and nonlinear feedback control gains of one-degree-of-freedom airfoil and their dimensionless counterparts, respectively
H_1, H_3	=	first- and third-order Volterra kernels
h, α, β	=	plunging, pitching, and flap displacements, respectively
I_α, I_β	=	mass moment of inertia per unit length of the wing-flap and of the flap about the elastic axis, and about the flap axis of rotation, respectively
k_h, k_α, k_β	=	stiffness parameters in plunging: torsional stiffnesses of the wing and flap about the elastic axis and about the flap axis of rotation, respectively
$\bar{k}$	=	reduced frequency, $\omega b/U_\infty$
L_a, L_b, L_c	=	aerodynamic lift, time-dependent external load, and active feedback control, respectively
$\boldsymbol{M}, \boldsymbol{K}, \boldsymbol{B}$	=	structural matrices
$\boldsymbol{M}_a, \boldsymbol{K}_a, \boldsymbol{B}_a$	=	aerodynamic matrices
m, μ	=	mass per unit of length and mass ratio ($\equiv m/2\rho_\infty b^2$), respectively
r_α, r_β	=	dimensionless radii of gyration of the wing flap, $(I_\alpha/mb^2)^{1/2}$, and of the flap, $(I_\beta/mb^2)^{1/2}$, respectively
S_α, χ_α	=	static unbalance about the elastic axis and its dimensionless counterpart, S_α/mb
S_β, χ_β	=	static unbalance about the flap axis of rotation and its dimensionless counterpart, S_β/mb
s	=	Laplace transform variable
T_i	=	Theodorsen's constants
t, σ	=	time and dummy time variables, respectively
$t_i, \tilde{t}_i$	=	time delay and dimensionless time delay, $t_i\omega_\alpha$, $i = \overline{1, 4}$, respectively
U_∞, V	=	freestream speed and its dimensionless counterpart $V = U_\infty/\omega_0 b$, respectively
$\boldsymbol{x}$	=	plunging, pitching, and flap displacement vector
$\zeta_\xi, \zeta_\alpha, \zeta_\beta$	=	structural damping ratios in plunging ($\equiv c_h/2m\omega_h$), pitching ($\equiv c_\alpha/2I_\alpha\omega_\alpha$), and flapping ($\equiv c_\beta/2I_\beta\omega_\beta$), respectively
ξ, α	=	plunging and pitching displacement quantities
ρ_∞	=	air density
τ	=	dimensionless time variable, tU_∞/b
$\phi(\tau)$	=	Wagner's function
$\omega_h, \omega_\alpha, \omega_\beta$	=	uncoupled frequencies in plunging, $(k_h/m)^{1/2}$, pitching, $(k_\alpha/I_\alpha)^{1/2}$, and flapping $(k_\beta/I_\beta)^{1/2}$, respectively

Subscripts

OL	=	open-loop
CL	=	closed-loop
$()'$	=	$\mathrm{d}()/\mathrm{d}t$

Introduction

IN recent years, the search for an enhanced response of aeroelastic systems has prompted increased interest in the incorporation of advanced control capabilities into their design. Their incorporation results in a closed-loop system and implies that the aeroelastic

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response is usually evaluated by a sensor and is fed back to an actuator that generates a force and/or moment. Related to this topic, excellent surveys are presented in Refs. 1 and 2, whereas analysis of stability properties of nonlinear aeroelastic systems and design of controllers for flutter suppression are considered in Refs. 3–6.

One of the main objectives in implementation of feedback control methodologies stems from the necessity of suppressing or even postponing the occurrence of flutter instability, its search being based on linear open/closed-loop aeroelastic systems. For evident reasons,[4–7] the study of aeroelastic systems in the postflutter range achievable within a nonlinear formulation presents considerable importance. In this sense, aeroelastic systems can experience stable or unstable limit cycles, in which cases the flutter boundary can be benign or catastrophic, respectively. The determination of the character of the flutter boundary depends on the type of Hopf bifurcation (HB): supercritical in the former case and subcritical in the latter one.

On the other hand, in the context of the control of aeroelastic systems, the unavoidable presence of a time delay between the controller and actuators should be accounted for.[7–9] In fact, the actuators may input energy at the moment when the controlled system does not need it.[7–10] These delays can be very detrimental in the sense of impairing the control performance and can even cause irregular motions, producing instability of the aeroelastic system. As indicated in Ref. 2, time delay is always present in digital control systems and tends to reduce the stability margins of carefully designed flutter-suppression systems. Time delays often occur in many other dynamical systems[11–16] and their existence constitutes a cause of instability and poor performance. However, there are cases where these delays are used to control chaotic motions[17] and if well designed, a time-delayed feedback control law can be used to convert the unstable limit cycle oscillation (LCO)[11] into a stable one. Within traditional feedback control models, there is no delay in the control. For all of these reasons, study of the stability and stabilization of linear time-delay systems has received considerable attention in recent years. However, to the best of the authors' knowledge, for nonlinear aeroelastic systems, the literature on these issues appears to be rather scarce. To address the implications of the time delay for linear/nonlinear feedback control and to render a problem that involves a great degree of complexity as simple and clear as possible, in this paper the open-loop aeroelastic system was considered to be linear. For this reason, this paper mainly deals with aeroelastic instability in conjunction with the presence of time delay in feedback control. To emphasize the importance and the possible detrimental effects of the time delay, the case of a pure plunging two-dimensional wing section is presented in full detail. This system can never become unstable, due to the presence of the aerodynamic damping, unless it is destabilized by the feedback loop control.

The methodology used in this work is based on Volterra series and indicial functions in conjunction with feedback control.[6–9] Volterra's functional series technique was proven to be an efficient tool in the analysis and solution of various nonlinear aeroelastic problems,[6] and its use can also be extended to the formulation of stability criteria for systems featuring time delays.

Background

The idea of incorporating time delay into feedback control stems from the fact that in aeroelasticity, as in other fields of engineering, biology, and physics, one often encounters dynamical systems that may be described as systems with memory or systems with delayed feedback.[8–17] In this context, considerable research has been done for more than three decades on various aspects of dynamical systems with delays in the state variables and/or control inputs.[18–24] Various stability criteria and numerical approaches have been presented in recent years (see Refs. 24–26 and references cited therein), and in that context pertinent applications have been reported.[8–11,17,27–29] In this sense, it is interesting to indicate Ref. 20, where an investigation of stability and chaos for wheel suspensions was presented. A stability analysis has been conducted in Refs. 10 and 29 for a linear, damped single-degree-of-freedom (SDOF) system with time delay involving displacement and velocity feedback controls. On the other hand, the problem of controlling unstable motion is vital in aeroelastic design of advanced flight vehicles. A few methodologies to achieve this goal have been devised. Among these there is a method developed by Ott et al.[18] based on the invariant manifold structure of unstable orbits and the one developed by Pyragas[19] based on time-delayed controlling forces.

For aeroelastic systems, time-delayed feedback was applied in Ref. 17 toward the control of the chaotic motion of a two-dimensional lifting surface with cubic pitching stiffness and linear viscous damping, using the feedback control method of Pyragas.[19]

The study of time-delayed feedback control for two-dimensional lifting surfaces has been presented in Refs. 8, 9, 12, and 17. Whereas in Ref. 8 only linear time-delay feedback control was considered, in Refs. 9, 12, and 17 both structural and aerodynamic nonlinearities were included in conjunction with linear and nonlinear time-delay feedback control. However, because of the evident complexity of the system, the effects of linear and nonlinear time-delay feedback control were obscured. For this reason, in this paper, only the effects of time delay in linear and nonlinear feedback control are considered. An inherent complexity in the delayed control systems stems from the fact that the associated characteristic equation, being transcendental, has an infinite number of roots, so that it is neither possible to solve for its roots nor to easily find approximate solutions.[8,9,12,19–24] In addition, aeroelastic systems incorporating feedback control capabilities can be extremely complex. For all these reasons, as a first step toward nonlinear analysis, the stability of linear differential–difference aeroelastic equations has to be studied.[7,8]

This study can provide useful information and also address some basic questions, such as whether aeroelastic stability is affected by the presence of delays in linear and nonlinear feedback control, and whether system stability is robust with respect to small variations of feedback gain.

As reported in Ref. 6, aeroelastic systems with multiple DOF, including structural and aerodynamic nonlinearities, can be investigated via a combined Volterra series[6,30–32] and indicial functions technique.[7] In Ref. 7 Volterra's series approach was applied to the open/closed-loop aeroelasticity of airfoils. It was shown that this method provides a good basis for a unified and efficient approach to nonlinear aeroelasticity problems. By performing a linear stability analysis of the aeroelastic system via the use of the first-order Volterra kernel, one can determine which class of orbits are accessible to time-delay feedback control methods. Explicit expressions for the critical time delay and control gains or the dependence of the transient behavior on the control parameters are derived. In this paper the stability boundary of a reduced-order open/closed-loop aeroelastic system incorporating nonlinear time-delayed feedback control is presented. The goals of designing such a system are, among others, controlling the aeroelastic response in the subcritical flight speed range, increasing the flutter speed, and converting the catastrophic flutter boundary into a benign one.

Analytical Developments

The first step toward the modeling of an open/closed-loop aeroelastic system with nonlinear time-delayed feedback control via the Volterra series approach is to determine the aeroelastic kernels. For the purpose of the present analysis, the approach presented in Refs. 6 and 7 has been modified. For an exhaustive treatment of the Volterra series concept applied to structural dynamics, the interested reader is referred to Ref. 32.

The aeroelastic kernels, including control effects, are derived in terms of structural parameters, unsteady aerodynamics, proportional (PFC) and velocity (VFC) feedback control gains, and feedback time delays. On this basis, the time histories and flutter boundary of the open/closed-loop delayed aeroelastic system are obtained. To this end, determination for each specific flight condition of the corresponding linear and nonlinear kernels of the Volterra series is required.[6,7] The open/closed-loop aeroelastic governing equation of an airfoil featuring plunging-pitching-flap deflection motion and subjected to external time-dependent loads can be expressed as

$$\boldsymbol{M}_s\ddot{\boldsymbol{x}}(t) + \boldsymbol{B}_s\dot{\boldsymbol{x}}(t) + \boldsymbol{K}_s\boldsymbol{x}(t) = (1/m)[\boldsymbol{F}_a(t) + \boldsymbol{F}_b(t)] + \boldsymbol{G}\boldsymbol{u}(t) \quad (1)$$

where $\boldsymbol{x}(t) = [h(t), \alpha(t), \beta(t)]^T$ and $\boldsymbol{u}(t)$ is the control input. For three DOF the control can consist of a torque applied at the flap.[33] The unsteady aerodynamic loads are represented by

$$\boldsymbol{F}_a(t) = \boldsymbol{M}_a\ddot{\boldsymbol{x}}(t) + \boldsymbol{B}_a\dot{\boldsymbol{x}}(t) + \boldsymbol{K}_a\boldsymbol{x}(t) + \boldsymbol{F}_c(t) \tag{2}$$

The significance of the other parameters is well known.[34,35] For the present case of time-delay control, a closed-loop system can be seen as an open-loop system where the transfer function includes the feedback control. In Eq. (1) the full state feedback control with delay can be expressed in the form

$$\boldsymbol{G}\boldsymbol{u}(t-\tau) = \boldsymbol{g}_p\boldsymbol{x}(t-\tau) + \boldsymbol{g}_v\dot{\boldsymbol{x}}(t-\tau) + \boldsymbol{g}_{\text{nc}}\boldsymbol{x}^3(t-\tau) \tag{3}$$

where $\boldsymbol{g}_p$, $\boldsymbol{g}_v$, and $\boldsymbol{g}_{\text{nc}}$ are the feedback gain matrices for the displacement, velocity, and the nonlinear control terms, respectively. Because the aeroelastic system incorporating feedback control forces and moments with time delays in the state feedback is of evident complexity, for a better understanding of the problem and of the present procedure, a simplified model has been adopted.

Delayed Aeroelastic System: Stability and Response

Some concepts related to the aeroelastic response and stability of one-DOF plunging airfoils in the presence of time delays between the sensing and the action of the actuator are presented next. It should be recalled (see, for example, Ref. 34) that because of aerodynamic damping, such a system cannot experience flutter instability. However, as will be shown, as a result of the time delay in linear feedback control, an instability can be induced and the instability boundary can be converted via nonlinear control from catastrophic to benign.

A one-DOF plunging airfoil is modeled as follows[6,7]:

$$m\ddot{h}(t) + c\dot{h}(t) + kh(t) = -L_a(t) + L_b(t) + L_c(t) \tag{4}$$

On the right-hand side of this equation, the unsteady aerodynamic lift is represented as

$$L_a(t) = C_{L\alpha}\rho U_\infty b\int_{-\infty}^{t}\phi(t-\sigma)\frac{\partial\dot{h}(\sigma)}{\partial\sigma}\,\mathrm{d}\sigma + \frac{1}{2}\rho C_{L\alpha}b^2\ddot{h} \tag{5}$$

The noncirculatory components of the unsteady aerodynamic load have been represented in terms of a convolution integral of the indicial Wagner function $\phi(\tau)$, where the added mass is associated with the term $\frac{1}{2}\rho C_{L\alpha}b^2\ddot{h}$.

For reasons already indicated, to highlight the implications of the time delay for linear and nonlinear control, the associated structural and aerodynamic models are considered as being linear. For an aeroelastic model in which structural nonlinearities were included, see for example Ref. 7. In addition, in Eq. (4) $L_b(\tau)$ denotes the external time-dependent load acting on the rigid wing counterpart, and $L_c(\tau)$ denotes the nonlinear feedback control force:

$$L_c(t) = g_p h(t-t_1) + g_v\dot{h}(t-t_2) - g_{\text{nc}}h^3(t-t_3) \tag{6}$$

Within the present work, the PFC and VFC have been supplemented by nonlinear PFC (with delays t_i, $i = \overline{1,3}$).

Letting $\tau = tU_\infty/b$ and $\xi = h/b$, the governing equation of the system with a nonlinear actuator control force featuring delay can be written in dimensionless form as

$$\underbrace{\xi'' + 2\zeta_\xi\frac{\bar{\omega}}{V}\xi' + \left(\frac{\bar{\omega}}{V}\right)^2\xi}_{\text{structural terms}} = \underbrace{-\frac{2}{\mu}\int_{-\infty}^{\tau}\phi(\tau-\sigma)\xi''\,\mathrm{d}\sigma - \frac{1}{\mu}\xi''}_{\text{aerodynamic terms}}$$
$$\underbrace{+\mathsf{g}_p\left(\frac{\bar{\omega}}{V}\right)^2\xi(\tau-\tau_1) + 2\zeta_\xi\mathsf{g}_v\frac{\bar{\omega}}{V}\xi'(\tau-\tau_2)}_{\text{linear control terms}}$$
$$\underbrace{-\mathsf{g}_{\text{nc}}\left(\frac{\bar{\omega}}{V}\right)^2\xi^3(\tau-\tau_3)}_{\text{nonlinear control term}} + \underbrace{L_b(\tau)}_{\substack{\text{time-dependent}\\\text{external load}}} \tag{7}$$

Evaluation of First- and High-Order Aeroelastic Kernels

Paralleling the procedure presented in Refs. 6 and 7, assuming a periodic external excitation of the form

$$L_b(t) = \sum_{j=1}^{n} X_j e^{s_j t} \tag{8}$$

the first- and high-order Volterra kernels of the aeroelastic system can be derived.

The identification of the nth order aeroelastic kernels is based on a general input in the form given by Eq. (8) and on the extraction, for the generic term of nth order, of the coefficients of

$$\prod_{i=1}^{n} X_i e^{s_i\tau}$$

This procedure was detailed in Ref. 7, where the expressions for the first three Volterra kernels of two-dimensional lifting surfaces have been explicitly derived.

Assuming a solution for the plunging displacement in the form $\xi(\tau) = H_{1_{\text{OL}}}(s)X_1e^{s\tau} + \cdots$, the first-order kernel $H_{1_{\text{OL}}}(s)$ characterizing the open-loop one-DOF aeroelastic system can be represented as

$$H_{1_{\text{OL}}}(s) = \left\{s^2 + 2\zeta_\xi(\bar{\omega}/V)S + (\bar{\omega}/V)^2 + (2/\mu)\left[\Phi(s) + \tfrac{1}{2}\right]s^2\right\}^{-1} \tag{9}$$

The first Volterra kernel of the closed-loop system $H_{1_{\text{CL}}}(s)$ is given by

$$H_{1_{\text{CL}}}(s) = \frac{H_{1_{\text{OL}}}(s)}{\left[1 + H_{1_{\text{OL}}}(s)\beta(s)\right]} \tag{10}$$

or, in explicit form, as

$$H_{1_{\text{CL}}}(s) = \left\{s^2 + 2\zeta_\xi(\bar{\omega}/V)s + (\bar{\omega}/V)^2 + (2/\mu)\left[\Phi(s) + \tfrac{1}{2}\right]s^2\right.$$
$$\left. + \mathsf{g}_p(\bar{\omega}/V)^2e^{-s\bar{\tau}_1} + 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V)se^{-s\bar{\tau}_2}\right\}^{-1} \tag{11}$$

where the feedback gains are taken in absolute value. Usually, these gains are negative in the LQG/LQR design methodologies. In the present case we assume that the feedback control is represented by $\beta(s)$; $\beta(s)$ can be one of the PFC or VFC gain, or a combination of these (CFC),[6] in conjunction with nonlinear proportional control gain.

For the specific case of the one-DOF airfoil with linear control presented in Eq. (11),

$$\beta(s) = \mathsf{g}_p(\bar{\omega}/V)^2e^{-s\bar{\tau}_1} + 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V)se^{-s\bar{\tau}_2} \tag{12}$$

From Eq. (10), when $\beta(s) = 0$, that is, in the case of the open loop, we have consistently $H_{1_{\text{OL}}} = H_{1_{\text{CL}}}$.

From a mathematical point of view, the closed-loop nonlinear aeroelastic system characterized by the first- and third-order kernels $H_{1_{\text{OL}}}(s)H_{3_{\text{OL}}}(s_1, s_2, s_3)$ and by the feedback gain $\beta(s)$ can be seen as an open-loop system described by the closed-loop aeroelastic kernels $H_{1_{\text{CL}}}(s)$, $H_{3_{\text{CL}}}(s_1, s_2, s_3)$ that are related to the kernels of the open-loop system and its control gains. It should be mentioned that the second-order kernel of the actual system is zero by virtue of the fact that no quadratic terms are involved in this system. As a result, only the first- and third-order kernels have to be considered toward determination of the nonlinear aeroelastic response and of the stability boundary. To obtain the third-order closed-loop nonlinear aeroelastic kernel $H_{3_{\text{CL}}}(s_1, s_2, s_3)$, we assume that the input can be expressed as

$$L_b(\tau) = \sum_{i=1}^{3} X_i e^{s_i\tau}$$

The output $\xi(\tau)$ can be written as

$$\xi(\tau) = \sum_{i=1}^{3} H_{1\mathrm{CL}}(s_i) X_i e^{s_i\tau} + \sum_{i=1}^{3} H_{1\mathrm{CL}}(s_i) X_i^2 e^{2s_i\tau} + \sum_{i=1}^{3} H_{1\mathrm{CL}}(s_i) X_i^3 e^{3s_i\tau} + 3H_{3\mathrm{CL}}(s_1, s_2, S_3) X_1 X_2 X_3 e^{(s_1+s_2+s_3)\tau} + \text{others} \tag{13}$$

Substituting the expression of $\xi(\tau)$ as given by Eq. (13) into Eq. (7), extracting the

$$\prod_{i=1}^{3} e^{s_i\tau}$$

term and using the expression of $H_{1\mathrm{CL}}(s_i)$, the closed-loop third-order aeroelastic kernel is obtained:

$$H_{3\mathrm{CL}}(s_1, s_2, s_3) = 2\mathrm{g}_{\mathrm{nc}}(\bar{\omega}/V)^2 H_{1\mathrm{CL}}(s_1) H_{1\mathrm{CL}}(s_2) H_{1\mathrm{CL}}(s_3) \times H_{1\mathrm{CL}}(s_1+s_2+s_3) e^{-(s_1+s_2+s_3)\tau} \tag{14}$$

In the context of dynamical systems modeled via Volterra series, it is a general property of the systems that all higher order kernels can be expressed in terms of lower order ones.

Stability Analysis and the Aeroelastic Stability Chart

For stability purposes, the aeroelastic system in the absence of external excitation, $L_b(\tau) = 0$, is considered. Without aerodynamic terms (which include time lags), and in the absence of control (i.e., $\mathrm{g}_p = \mathrm{g}_v = 0$), the system is dissipative with two finite stable characteristic roots (poles) on the left half of the complex plane. However, for the aeroelastic system with feedback delayed control ($\tilde{\tau}_j > 0$; $j = 1, 2, 3$), the two finite stable roots are supplemented by other finite stable roots (whose number depends on the aerodynamic model), and, due to the presence of $e^{-s\tilde{\tau}}$ in the characteristic equation, by an additional infinite number of roots.

The conditions that guarantee the stability of the delayed system have been studied by Pontryagin[22] and applied toward the stability of time-delayed feedback control systems (see, e.g., Refs. 8, 9, 12, and 36). In the present aeroelastic analysis, Pontryagin's approach[22] in conjunction with Stépán's theorems[25] has been adopted.

As proved in Ref. 24, the stability or instability of delayed aeroelastic systems analyzed using the concept of a retarded functional differential equation (RFDE) depends on the presence of zeros with positive real parts of the characteristic equation, that is, on the presence of the p-zeros.

Whereas the characteristic equation of an ordinary differential equation is a polynomial and the conditions for absence of p-zeros for polynomials are given by the well-known Routh–Hurwitz theorem, for RFDEs the characteristic equation can be expressed in quasi-polynomial form as[24]

$$D(z) = \sum_{l=0}^{m} \sum_{j=1}^{r} a_{lj} z \exp(b_j z) \tag{15a}$$

Although analytically more complex, the Routh–Hurwitz criterion can be applied to Eq. (15a). For stability evaluations, the characteristic equation of the unforced linearized counterpart of Eq. (7) can be written as

$$D(s) = 1/H_{1\mathrm{CL}}(s) = 0 \tag{15b}$$

In the absence of time delay, the following relation is valid:

$$D(s) = s^2 + 2\zeta_\xi(\bar{\omega}/V)s + (\bar{\omega}/V)^2 + (2/\mu)\left[\Phi(s) + \tfrac{1}{2}\right]s^2 + \mathrm{g}_p(\bar{\omega}/V)^2 + 2\zeta_\xi \mathrm{g}_v(\bar{\omega}/V)s = 0 \tag{15c}$$

Since $1 + (2/\mu)[\Phi(s) + \frac{1}{2}] > 0$, the stability conditions are obtained by imposing $\mathrm{g}_v > -1$ and $\mathrm{g}_p > -1$.

Notice that the characteristic roots (i.e., poles) of Eq. (15a) are of the form $s = a + i\omega$. As a particular case, if $\mathrm{g}_p = \mathrm{g}_v = \mathrm{g}$, and $\tilde{\tau} = \tilde{\tau}_j$, the following relation holds valid:

$$g = \frac{\left|\left\{1 + (2/\mu)\left[\Phi(s) + \frac{1}{2}\right]\right\}(V/\bar{\omega})^2 s^2 + 2\zeta_\xi(V/\bar{\omega})s + 1\right|}{|1 + 2\zeta_\xi(V/\bar{\omega})s|} e^{a\tilde{\tau}} \tag{16a}$$

It is readily seen that for the uncontrolled system, implying $\mathrm{g}_p = \mathrm{g}_v = 0$, the characteristic equation (15) has four finite stable poles in the complex plane that are obtained by solving the equation

$$\left|\left\{1 + (2/\mu)\left[\Phi(s) + \tfrac{1}{2}\right]\right\}(V/\bar{\omega})^2 s^2 + 2\zeta_\xi(V/\bar{\omega})s + 1\right| = 0 \tag{16b}$$

and all remaining poles are at $a = -\infty$. For $\mathrm{g}_p = \mathrm{g}_v = \infty$, there are poles at $s = 0, -1, -82.36$, and -12.89, and the remaining poles are at $a = +\infty$.

For equal time delays, via time transformation with respect to the delay, that is, replacing $s\tilde{\tau}$ with $\hat{s}$, Eq. (15) can be further simplified as

$$D(\hat{s}) = \hat{s}^2 + 2\zeta_\xi(\bar{\omega}/V)\hat{s}\tilde{\tau} + (\bar{\omega}/V)^2\tilde{\tau}^2 + (2/\mu)\Phi(\hat{s}/\tilde{\tau})\hat{s}^2 + (1/\mu)\hat{s}^2 + \mathrm{g}_p(\bar{\omega}/V)^2\tilde{\tau}^2 e^{-\hat{s}} + \mathrm{g}_v 2\zeta_\xi(\bar{\omega}/V)\hat{s}\tilde{\tau}e^{-\hat{s}} = 0 \tag{17}$$

The stability of Eq. (17) will be studied via Stépán's analytical method.[20,25] Consistent with it, upon denoting $\rho_1 \geq \cdots \geq \rho_r \geq 0$ and $\sigma_1 \geq \cdots \geq \sigma_s = 0$, the nonnegative real zeros of

$$R(\omega) = \mathrm{Re}[D(i\omega)] = (-1)^m \omega^n + \mathrm{O}(\omega^n) \tag{18a}$$

$$S(\omega) = \mathrm{Im}[D(i\omega)] = \mathrm{O}(\omega^n) \tag{18b}$$

the trivial solution $\xi = 0$ of the system is exponentially asymptotically stable, if and only if $R(0) > 0$; $n = 2m$ (n is the order of the system and m is integer); $S(\rho_k) \neq 0$ for $k = 1, \ldots, r$; and

$$\sum_{k=1}^{r} (-1)^k \operatorname{sgn} S(\rho_k) = (-1)^m m \tag{19}$$

Similar conditions of stability are defined for systems where $n = 2m + 1$; see Ref. 20. For the present case, replacing $\Phi(\hat{s})\hat{s}^2 \Rightarrow C(\hat{s})\hat{s}\tilde{\tau}$ and $\hat{s} \Rightarrow i\omega$, where $C(\bar{k})$ ($\equiv F(\bar{k}) + iG(\bar{k})$) is the Theodorsen's function, and considering the real and imaginary parts of Eq. (17), one obtains

$$R(\omega) = -\omega^2 + (\bar{\omega}/V)^2\tilde{\tau}^2 - (2/\mu)\omega\tilde{\tau}G(\bar{k}/\tilde{\tau})\omega - (1/\mu)\omega^2 + \mathrm{g}_p(\bar{\omega}/V)^2\tilde{\tau}^2\cos\omega + 2\zeta_\xi(\bar{\omega}/V)\mathrm{g}_v\tilde{\tau}\omega\sin\omega \tag{20a}$$

$$S(\omega) = 2\zeta_\xi(\bar{\omega}/V)\tilde{\tau}\omega + (2/\mu)\omega\tilde{\tau}F(\bar{k}/\tilde{\tau}) - \mathrm{g}_p(\bar{\omega}/V)^2\tilde{\tau}^2\sin\omega + 2\zeta_\xi(\bar{\omega}/V)\mathrm{g}_v\tilde{\tau}\omega\cos\omega \tag{20b}$$

The trivial solution of Eq. (7) is exponentially asymptotically stable[8,9,20] if and only if

$$\mathrm{g}_p > -1 \tag{21a}$$

$$\tilde{\tau} < 2\zeta_\xi(V/\bar{\omega})(\mathrm{g}_v/\mathrm{g}_p) \tag{21b}$$

$$\mathrm{g}_p < (\sigma/\tau)(V/\bar{\omega})^2\left\{[\sigma/\tau - (\bar{\omega}/V)^2(\tilde{\tau}/\sigma) + (1/\mu)(\sigma/\tilde{\tau})]\cos\sigma + 2\zeta_\xi(\bar{\omega}/V)\sin\sigma + (2/\mu)(G\cos\sigma + F\sin\sigma)\right\} \tag{21c}$$

Here, σ is the smallest positive zero of the equation

$$S(\sigma) = 2\zeta_\xi(\bar{\omega}/V)\tilde{\tau}\sigma + (2/\mu)\sigma\tilde{\tau}F - \mathrm{g}_p(\bar{\omega}/V)^2\tilde{\tau}^2\sin\sigma + 2\zeta_\xi(\bar{\omega}/V)\mathrm{g}_v\tilde{\tau}\sigma\cos\sigma = 0 \tag{22}$$

where $\sigma \in [0, (\pi/2)]$. The proof of Eq. (17) is given next.

The inequality $\mathrm{g}_p > -1$ is obtained from the condition

$$R(0) = (\bar{\omega}/V)^2\tilde{\tau}^2 + \mathrm{g}_p(\bar{\omega}/V)^2\tilde{\tau}^2 > 0 \tag{23}$$

Considering the smallest positive root σ of Eq. (20b), one obtains that $S(\omega) > 0$, $\omega \in (0, \sigma)$ if and only if

$$2\zeta_\xi(\bar{\omega}/V)\omega + (2/\mu)\omega F - \mathsf{g}_p(\bar{\omega}/V)^2\tilde{\tau}\sin\omega + 2\zeta_\xi(\bar{\omega}/V)\mathsf{g}_v\omega\cos\omega > 0 \tag{24}$$

which yields

$$\mathsf{g}_p(\bar{\omega}/V)^2\tilde{\tau}\sin\omega < 2\zeta_\xi(\bar{\omega}/V)\omega + (2/\mu)\omega F + 2\zeta_\xi(\bar{\omega}/V)\mathsf{g}_v\tau\omega\cos\omega \tag{25}$$

Since the first two terms of Eq. (25) are always positive, one can conclude that the condition $\tilde{\tau} < 2\zeta_\xi(V/\bar{\omega})(\mathsf{g}_v/\mathsf{g}_p)$ is necessary to fulfill the inequality (25). In addition, for the smallest positive root σ, we can state that

$$R(\sigma) = -\sigma^2 + (\bar{\omega}/V)^2\tilde{\tau}^2 - (2/\mu)\sigma\tilde{\tau}G\sigma - (1/\mu)\sigma^2 + \mathsf{g}_p(\bar{\omega}/V)^2\tilde{\tau}^2\cos\sigma + 2\zeta_\xi(\bar{\omega}/V)\mathsf{g}_v\tilde{\tau}\sigma\sin\sigma < 0 \tag{26a}$$

After straightforward algebraic manipulations one obtains

$$\mathsf{g}_p\cos^2\sigma < \left\{[(V/\bar{\omega})(\sigma/\tilde{\tau})]^2 - 1 + (2/\mu)(\sigma/\tilde{\tau})(V/\bar{\omega})^2 G - (1/\mu)[(V/\bar{\omega})(\sigma/\tilde{\tau})]^2 - 2\zeta_\xi[(V/\bar{\omega})(\sigma/\tilde{\tau})]\mathsf{g}_v\sin\sigma\right\}\cos\sigma \tag{26b}$$

and using standard trigonometric relationships, Eq. (26b) in conjunction with Eq. (25), rewritten in the form

$$\mathsf{g}_p\sin^2\sigma < \left\{2\zeta_\xi[(V/\bar{\omega})(\sigma/\tilde{\tau})] + (2/\mu)(\sigma/\tilde{\tau})(V/\bar{\omega})^2 F + 2\zeta_\xi[(V/\bar{\omega})(\sigma/\tilde{\tau})]\mathsf{g}_v\sigma\cos\sigma\right\}\sin\sigma \tag{27}$$

reduces to Eq. (21c). The condition (19), where $m = 1$, is also fulfilled.

The present approach for determining the stability domain of a delayed aeroelastic system has a certain similarity to Theodorsen's method for determining the flutter speed by plotting the real and imaginary parts of the flutter determinant in conjunction with consideration of a real ω. The former approach reduces to the latter in the case of zero time delay.

The stability chart of the aeroelastic system described by Eq. (7) with respect to the feedback gains and the time delay can be constructed using Stépán's theorem and the D-subdivision method.[12] The method of D-subdivision is applied to determine the conditions under which the quasi-polynomial $D_c(s)$ has no p-zeros.[25,37,38]

From the preliminary findings it appears that the closed-loop stability boundary is strongly affected by the velocity feedback control, especially in the case of the time delay in the control. In this sense, in the presence of delays, the velocity feedback control can be more detrimental than the proportional feedback control. This does not imply that the controller has no robustness to modeling errors or disturbances, but, in the design of the feedback control system, it is necessary to identify the stability boundary and its variation with the feedback gains and the time delay. This will become more evident from the results displayed in Results and Discussion.

As remarked in Refs. 8 and 9, because the quasi-polynomial is a continuous function of its parameters, one can construct the subdivision of the coefficient's space by hypersurfaces, the points of which are quasi-polynomials with at least one imaginary root. In addition, as proven in Ref. 25, with the variation of the quasi-polynomial parameters the number of p-zeros may change only by passage of some zeros through an imaginary axis, and for all points of every domain of D-subdivision the number of quasi-polynomial p-zeros will be the same.

Hopf Bifurcation Analysis

The Hopf bifurcation (HB) of the previously described aeroelastic systems will be presented next. Based on Refs. 20 and 39, assuming for the sake of simplicity equal time-delays $\tau_1 = \tau_2 = \tau_3 = \tilde{\tau}$, expanding the nonlinear time-delayed feedback control L_c into Taylor series and discarding the terms containing the nonlinear time delays yields

$$L_c(t) = -g_v\tilde{\tau}\ddot{h}(t) + (g_v - g_p\tilde{\tau})\dot{h}(t) + g_p h(t) + g_{\mathrm{nc}}h^3(t) - 3g_{\mathrm{nc}}\tilde{\tau}h^2(t)\dot{h}(t) + \mathrm{O}(\tilde{\tau}^2) \tag{28}$$

As a result, the aeroelastic governing equation is rewritten in the form

$$m\ddot{h}(t) + c\dot{h}(t) + kh(t) + g_v\tilde{\tau}\ddot{h}(t) - (g_v - g_p\tilde{\tau})\dot{h}(t) - g_p h(t) - g_{\mathrm{nc}}h^3(t) + 3g_{\mathrm{nc}}\tilde{\tau}h^2(t)\dot{h}(t) = -C_{L\alpha}\rho bU_\infty \times \int_{-\infty}^{t}\phi(t-\sigma)\ddot{h}(t)\,\mathrm{d}\sigma - \tfrac{1}{2}\rho C_{L\alpha}b^2\ddot{h}(t) + L_c(t) \tag{29}$$

and in dimensionless form as

$$\xi'' + 2\zeta_\xi\frac{\bar{\omega}}{V}\xi' + \left(\frac{\bar{\omega}}{V}\right)^2\xi = -\frac{2}{\mu}\int_{-\infty}^{\tau}\phi(t-\sigma)\xi''\,\mathrm{d}\sigma - \frac{1}{\mu}\xi'' - 2\zeta_\xi\mathsf{g}_v\frac{\bar{\omega}}{V}\tilde{\tau}\xi'' + \left[2\zeta_\xi\mathsf{g}_v\frac{\bar{\omega}}{V} - \mathsf{g}_p\left(\frac{\bar{\omega}}{V}\right)^2\tilde{\tau}\right]\xi' + \mathsf{g}_p\left(\frac{\bar{\omega}}{V}\right)^2\xi + \mathsf{g}_{\mathrm{nc}}\left(\frac{\bar{\omega}}{V}\right)^2\xi^3 - 3\mathsf{g}_{\mathrm{nc}}\left(\frac{\bar{\omega}}{V}\right)^2\tilde{\tau}\xi^2\xi' \tag{30}$$

For the reasons already indicated, the gains are taken in absolute value. This yields the following approximate nonlinear equation:

$$\left(1 - 2\zeta_\xi\mathsf{g}_v\frac{\bar{\omega}}{V}\tilde{\tau} - \frac{1}{\mu}\right)\xi'' + \left[2\zeta_\xi\frac{\bar{\omega}}{V} + 2\zeta_\xi\mathsf{g}_v\frac{\bar{\omega}}{V} - \mathsf{g}_p\left(\frac{\bar{\omega}}{V}\right)^2\tilde{\tau}\right]\xi' + \left(\frac{\bar{\omega}}{V}\right)^2(1+\mathsf{g}_p)\xi + \mathsf{g}_{\mathrm{nc}}\left(\frac{\bar{\omega}}{V}\right)^2(\xi^3 - 3\tilde{\tau}\xi^2\xi') + \frac{2}{\mu}\int_{-\infty}^{\tau}\phi(\tau-\sigma)\xi''\,\mathrm{d}\sigma = 0 \tag{31}$$

Stability Examination

The zero solution of Eq. (31), for the system in steady flow, is exponentially asymptotically stable[8,22] if

$$\mathsf{g}_p > -1 \tag{32a}$$

$$1 - 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V)\tilde{\tau} > 0 \tag{32b}$$

$$2\zeta_\xi(\bar{\omega}/V) + 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V) - \mathsf{g}_p(\bar{\omega}/V)^2\tilde{\tau} > 0 \tag{32c}$$

The HB occurs in the following conditions:

$$\mathsf{g}_p/(1+\mathsf{g}_v) = 2\zeta_\xi/(\bar{\omega}/V)\tilde{\tau} \tag{33a}$$

$$\mathsf{g}_v < 1/2\zeta_\xi(\bar{\omega}/V)\tilde{\tau} \tag{33b}$$

$$\mathsf{g}_p > -1 \tag{33c}$$

and it is supercritical or subcritical if

$$\mathsf{g}_{\mathrm{nc}} < 0 \qquad \text{or} \qquad \mathsf{g}_{\mathrm{nc}} > 0 \tag{34}$$

respectively. For the problem at hand, the proof of the HB condition (34) is given next. The nonlinear Eq. (31) can be linearized about $\xi = 0$:

$$[1 - 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V)\tilde{\tau}]\xi'' + \left[2\zeta_\xi(\bar{\omega}/V) + 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V) - \mathsf{g}_p(\bar{\omega}/V)^2\tilde{\tau}\right]\xi' + (\bar{\omega}/V)^2(1+\mathsf{g}_p)\xi = 0 \tag{35}$$

and its characteristic polynomial is given by

$$[1 - 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V)\tilde{\tau}]s^2 + \left[2\zeta_\xi(\bar{\omega}/V) + 2\zeta_\xi\mathsf{g}_v(\bar{\omega}/V) - \mathsf{g}_p(\bar{\omega}/V)^2\tilde{\tau}\right]s + (\bar{\omega}/V)^2(1+\mathsf{g}_p) = 0 \tag{36}$$

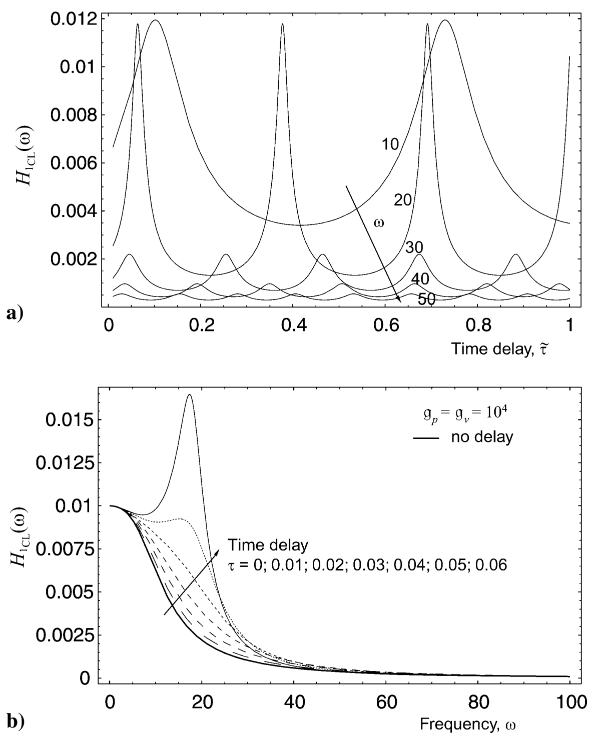

Fig. 1 First-order aeroelastic kernel: a) H_1 vs ω: influence of the time delay and b) H_1 vs τ: influence of the frequency.

Assuming that $g_v < V/2\zeta_\xi \bar{\omega}\tilde{\tau}$ and $g_v = (\bar{\omega}\tilde{\tau}/2\zeta_\xi V)g_p - 1$, then, as a necessary requirement for the Hopf bifurcation, a pair of purely imaginary roots of the characteristic equation on the critical boundary should exist. For the present case

$$s_{1,2} = \pm i\beta \tag{37a}$$

where

$$\beta = \sqrt{\frac{1 + g_p}{(V/\bar{\omega})^2 + 2\zeta_\xi (V/\bar{\omega})\tilde{\tau} - \tilde{\tau}^2 g_p}} \tag{37b}$$

By defining the new variables $y_1 = \xi$ and $y_2 = \xi'/\beta$ the Poincaré normal form is obtained:

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & -\beta \\ -\beta & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f_{30} y_1^3 + f_{21} y_1^2 y_2 \end{bmatrix} \tag{38}$$

where

$$f_{30} = -(1/\beta^3)(1 + g_p)g_{\mathrm{nc}} \tag{39a}$$

$$f_{21} = 3\tilde{\tau}(1/\beta^2)(1 + g_p)g_{\mathrm{nc}} = -3\tilde{\tau}\beta f_{30} \tag{39b}$$

The type of HB occurring at a selected time delay can be determined using the center manifold theorem. Specifically, the sign of the quantity L, referred to as the Lyapunov first quantity,[11,40,41] where

$$\mathsf{L} = \tfrac{1}{8} f_{21} \tag{40}$$

determined at the critical time delay, defines, when $\mathsf{L} < 0$ and $\mathsf{L} > 0$, the benign boundary (supercritical HB) and the catastrophic one

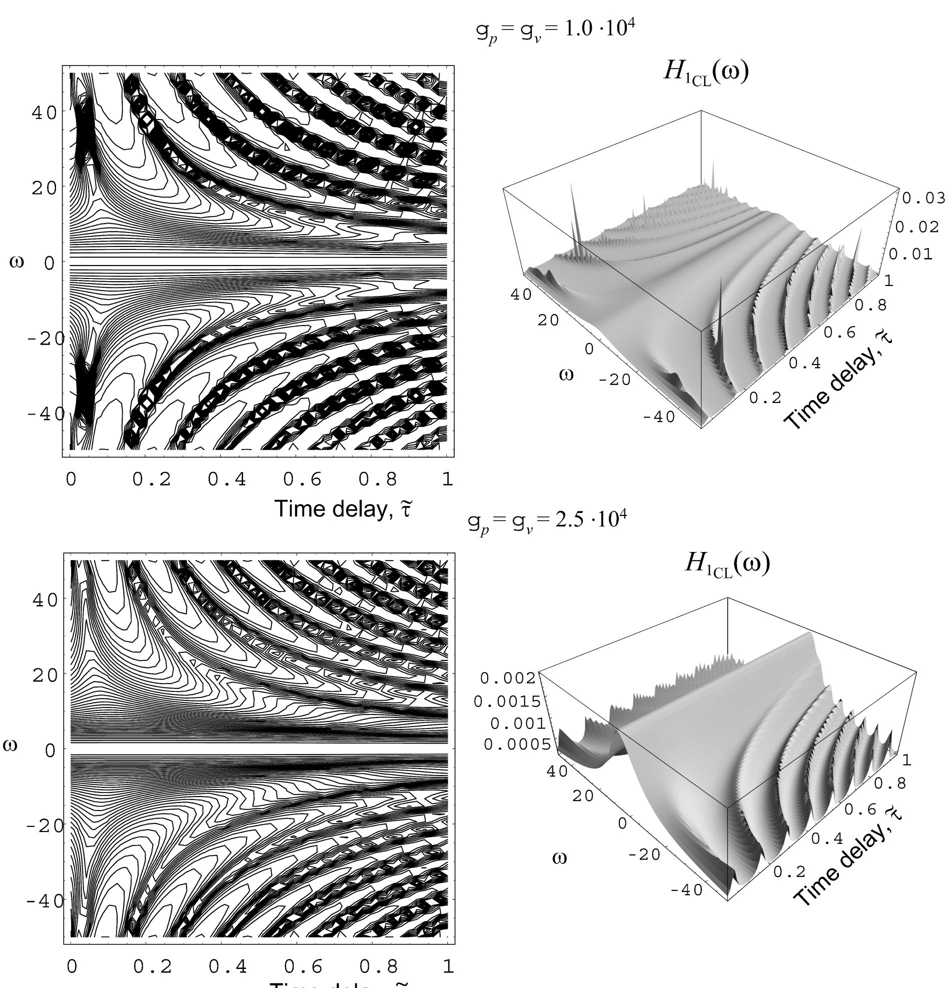

Fig. 2 Three-dimensional and contour plots of the first-order kernel. Variation of the frequency vs time delay for two values of the proportional and velocity feedback gains.

(subcritical HB), respectively. This implies that

$$f_{21} < 0 \Rightarrow \mathsf{g}_{\mathrm{nc}} < 0 \Rightarrow \text{supercritical HB} \tag{41a}$$

$$f_{21} > 0 \Rightarrow \mathsf{g}_{\mathrm{nc}} > 0 \Rightarrow \text{subcritical HB} \tag{41b}$$

In this case, the nonlinear feedback gain is an inherent part of the Lyapunov first quantity (e.g., Refs. 11 and 21) and its sign decides upon the character of the stability boundary. In Ref. 21 the methodology based on the Lyapunov quantity is presented in the context of a two-dimensional lifting surface without time delay, whereas in Ref. 11 the time delay has been included to study the stability of a supersonic two-dimensional lifting surface in the presence of time-delayed feedback control.

Results and Discussion

The dimensionless parameters for the simulation are $\mu = m/\pi\rho b^2$, $\bar{\omega} = \omega_h/\omega_0$, $\zeta_\xi = c/2m\omega_h$, $\mathsf{g}_p = \xi_p/k$, $V = U_\infty/\omega_0 b$, $\omega_h^2 = k/m$, $\mathsf{g}_v = \xi_v/c$, and $\mathsf{g}_{\mathrm{nc}} = g_{\mathrm{nc}}b^2/k$, and the airfoil parameters are $b = 1$ ft, $\zeta = 0.008$, $m = 10$ slug/ft $\omega_h = 60$ rad/s, $\rho_\infty = 0.0318$ slug/ft^3, $c = 2m\omega\zeta$, $k = \omega^2 m$, and $C_{L\alpha} = 2\pi$. In Figs. 1a and 1b, for selected values of the proportional and velocity feedback gains and of the dimensionless frequency ω/ω_h, the effects of the time delay on the first-order kernel is presented. It clearly appears that, due to the time delay, there is a shift of the peak location in the first-order Volterra kernel. This implies that the time delay can have detrimental or beneficial effects on the behavior of the aeroelastic system. In Figs. 2 and 3, three-dimensional and contour plots of the first- and third-order kernels against the frequency and the time delay are depicted for two values of the proportional and velocity feedback gains and two values of the time delay, respectively. These plots emphasize the effects of the time delay and of the feedback gains on Volterra kernels. In this sense, it clearly appears that larger values of the feedback gains produce lower peaks of the first-order kernel (Fig. 2) and that different values of the time delay can have a complex effect on the third-order kernel (Fig. 3). In addition, it is worth noting the peculiar wavy shape of the first-order kernel. This suggests that in the design of time-delay feedback control of aeroelastic systems, it is necessary to have combinations of the frequency and time delay such that the design point not lie on one of the crests of the wave, where large-amplitude response and instability behavior are expected to occur. The presence of new peaks in the third-order kernel appearing at higher values of the time delay clearly emphasize that there are time delays that can detrimentally affect the stability of a system, whereas for other ones a better stability behavior can be reached. For a better understanding of these issues, two-dimensional and three-dimensional stability charts are presented next.

The two-dimensional stability chart for this aeroelastic system is shown in Fig. 4. In this figure, there is represented the region in the $\{\mathsf{g}_p, \mathsf{g}_v\}$ parameter space where the roots of the characteristic equation of the system have zero real parts and the values of the geometrical parameters have been locked. Other stability regions can be drawn in the $\{\mathsf{g}_p, \mathsf{g}_v\}$ parameter space, but these are of no practical importance. From Fig. 4 one can remark that the domain of stability/instability is more sensitive to the variations of g_v than to

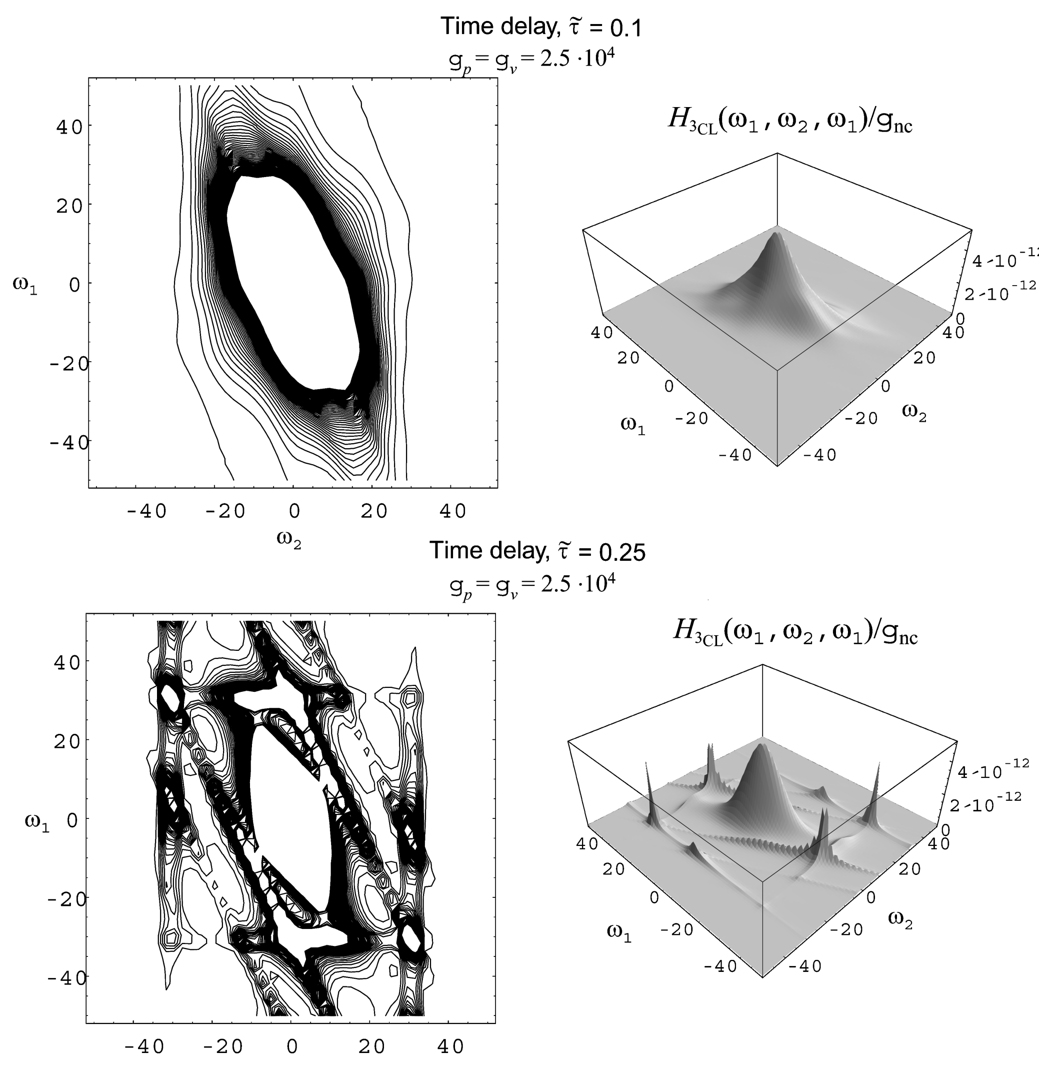

Fig. 3 Three-dimensional and contour plots of the third-order kernel. Variation of the frequency for two values of time delay.

those of g_p, implying that the velocity feedback control has a more powerful influence on the stability boundary than the proportional feedback control. On the other hand, if no time delays are present, independent of the values of the feedback gains, the stable parameter space is the complete positive quadrant of the $\{g_p, g_v\}$ parameter plane. This implies that, in the absence of time delay, the system is always stable.

Whereas, within a linear analysis, one can predict the instability boundary, the nonlinear analysis provides an insight into the character, benign or catastrophic, of the stability boundary. For this purpose an auxiliary stability chart (Fig. 4), based on Eqs. (33), makes possible the prediction of the type of Hopf bifurcation (i.e., supercritical or subcritical). For small time delays, this auxiliary plot supplies pertinent information related to the nature of the HB. However, because of the approximation used in Taylor series expansion, numerical instabilities (the horizontal lines in Fig. 4) are likely to occur. For this reason the auxiliary plot is used only to determine the HB conditions and not for a full examination of the nonlinear system.

On the basis of the HB condition and of Fig. 4, it can be seen that the HB occurs on the three-dimensional envelope of the parametric domain depicted in Fig. 5. Therefore, if the parameter point is located in the inner domain of the envelope, the system is stable, and if the parameter point is in the outer domain, the system features a supercritical or subcritical HB, depending of the sign of the nonlinear control gain, negative or positive, respectively.

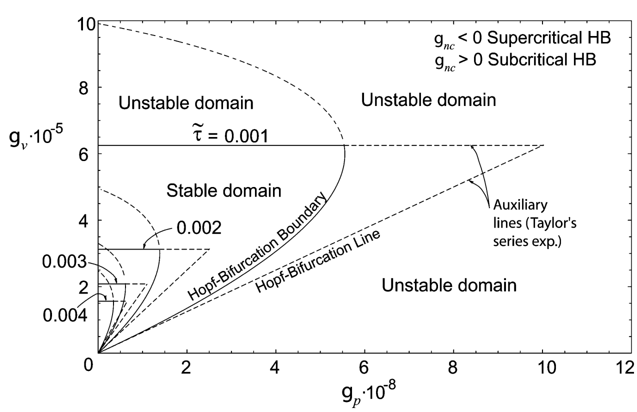

Fig. 4 Stability charts of the full and approximate (Taylor's expansion) closed-loop delayed nonlinear aeroelastic system.

Numerical Validation of Analytical Predictions

The absence of studies on this problem prevents the validation of the present findings. However, a numerical validation of analytical predictions has been carried out in this paper.

In Fig. 6 the nonlinear closed-loop aeroelastic responses of the one-DOF airfoil for $g_{nc} = -10$ (implying a supercritical HB) and $g_{nc} = 10$ (implying a subcritical HB) are presented. The onset of LCO obtained via Volterra series and the LCO behavior obtained via numerical integration are in full agreement with the analytical predictions of Eqs. (41a) and (41b).

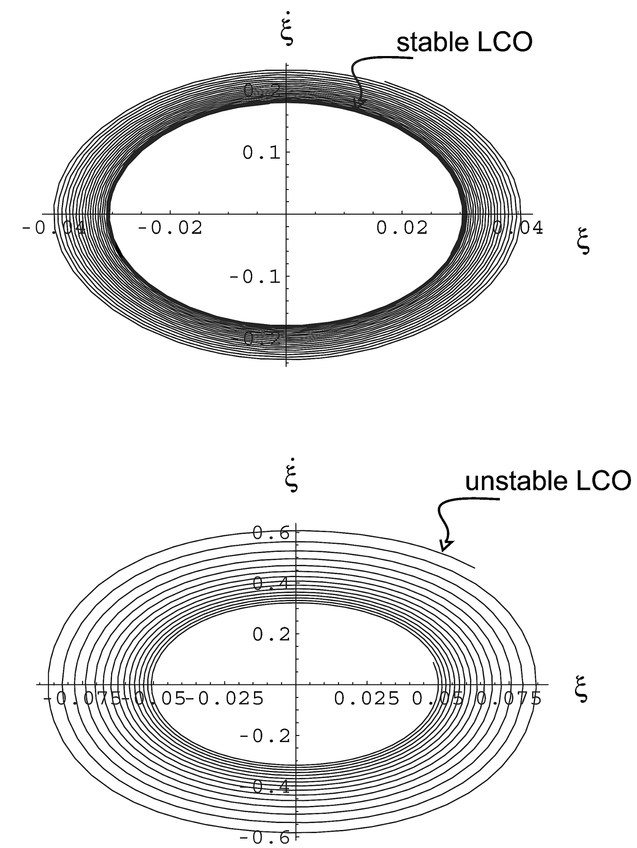

Fig. 6 Stable and unstable LCO for the plunging airfoil with active feedback delayed control law ($\tilde{\tau} = 0.01$).

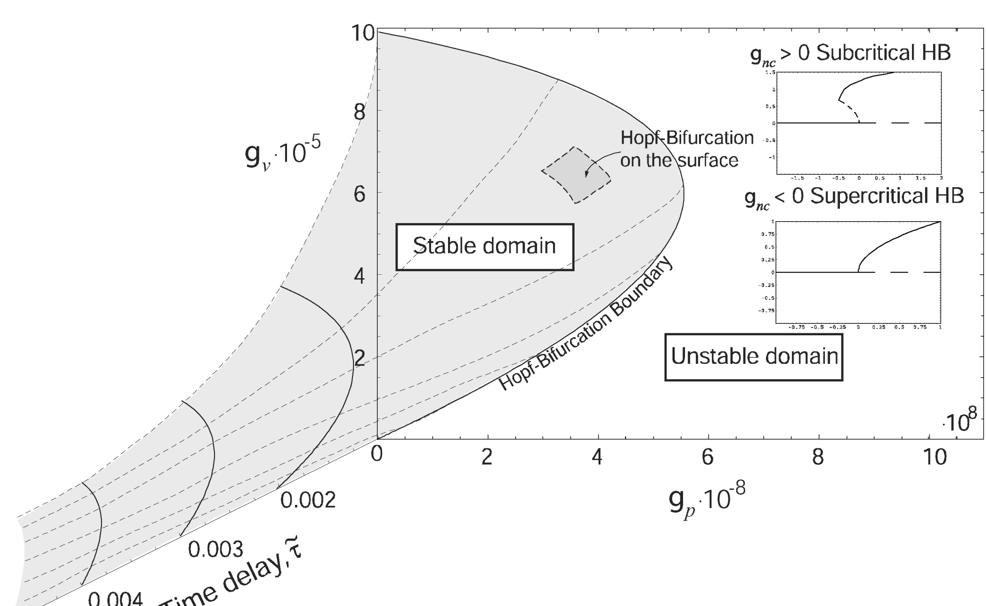

Fig. 5 Three-dimensional stability chart of the plunging airfoil. Effect of the time delay and of the control gains.

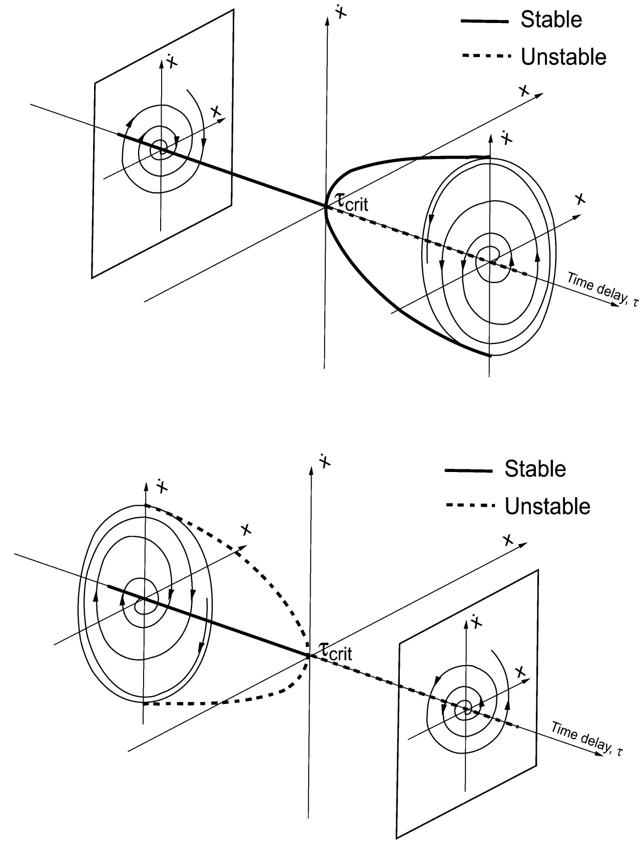

Fig. 7 Pictorial representation of two possible scenarios: supercritical (top) and subcritical (bottom) Hopf bifurcations occurring at the critical time delay.

Figure 7 illustrates the cases of supercritical and subcritical HB. In the case of supercritical HB, orbitally asymptotically stable periodic motion is reached for $\tau > \tau_{\mathrm{crit}}$. On the other hand, in the case of subcritical HB, unstable periodic motion exists around the stable stationary motion for $\tau < \tau_{\mathrm{crit}}$. In this sense, if the aeroelastic system exhibits variable time-delay but fixed gains, there will be a critical value of the time-delay τ_{crit} for which the Hopf bifurcation is encountered. This emphasizes the fact that this one-DOF plunging airfoil, even if it does not experience flutter, can encounter the instability generated by the presence of the time delay in the nonlinear feedback control. The obtained results emphasize the fact that linear and nonlinear time-delay feedback controls should be implemented in aeroelastic systems with caution, having in view that, depending on the magnitude of the time delay, these can produce beneficial or detrimental effects.

These results are in full agreement with ones in Ref. 11, where a procedure based on center manifold and normal forms was applied for a plunging-pitching airfoil.

Conclusions

A number of issues related to the effect of linear and nonlinear time-delay feedback control of two-dimensional lifting surface aeroelasticity have been discussed. In this context, a detailed analysis of the instability induced by the time delay in the linear feedback control of a one-DOF aeroelastic system was considered, and the possibility of rendering this instability boundary a benign one via nonlinear time-delay feedback control has been put into evidence. The complex phenomena reported here can stimulate further research on more general two- and three-dimensional aeroelastic systems. Their understanding can lead to a more reliable design of advanced aerospace vehicles. Nevertheless, the model presented here, based on the determination of higher order Volterra kernels and Hopf bifurcation maps, can be used to determine the aeroelastic response and flutter boundary of open/closed-loop aeroelastic systems. With the incorporation of structural and aerodynamic nonlinearities into the system, LCO and chaos are expected to occur; however, the analysis presented here unveils possible detrimental and beneficial effects of linear and nonlinear time delay in the feedback controls.

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